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The inverse-square law of force and its spatial energy distribution

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Abstract. Using generalised mathematical considerations the inverse-square law of force is shown to imply specific spatial energy distributions relative to the interacting bodies. The retardation effects associated with energy redeployment when the bodies are in motion are examined. It is found that, as applied to the gravitational interaction between sun and planet and provided there is no discontinuity in the spatial energy distribution, retardation will give a law of motion conforming with Einstein's law of gravitation. A necessary condition is that the energy in transit in the field system is ineffective in determining force for a retardation period equal to the time required for a photon to travel from one body to the field and then return from the field to the other body. The implication is that gravitation could be a quantum interaction which assures causality and balance of action and reaction by this dual photon exchange interaction.

1. Introduction

Much of accepted field theory is based upon the experimental foundation of Coulomb's law and the observed propagation of electromagnetic waves. Maxwell's equations provide a well tested starting point in field theory, and are all the more secure because they have not needed to be modified by the requirements of the theory of relativity. This property of Maxwell's equations is linked with their linearity. The fields they represent can be superimposed and added vectorially. Had the equations contained second-order terms, signifying energy density parameters, then the impact of relativity may well have been a different story.

In contrast, Coulomb's law or Newton's law of gravitation both depend upon interactions which are second order in this sense and both present difficulties when extended to relate to dynamic interactions. Energy has an enigmatic role in field theory, particularly when one tries to analyse the energy radiation from individually accelerated charge.

Our object in this paper is to approach the general problem of the dynamic interaction of bodies which are subject to an inverse-square of distance law of mutual force, but without adopting any field hypothesis. We will be guided by Brillouin (1970), who endorsed a statement by Heaviside: 'To form any notion at all of the flux of gravitational energy, we must first localise the energy.' Accordingly, our sole consideration will be the energy we associate with the interaction, taking energy as a scalar quantity and so avoiding the complications of the vector field.

2. General energy formula

Our task is to derive a general formula $f(r)$ for the distribution $\partial E/\partial r$ denoting the element of energy ∂E distant r from a body A and contained within a spherical shell of thickness ∂r centred on that body. We know that the body A is urged towards a second body B, distant x from A, by a force given by

$$F = K/x^2. \tag{1}$$

K is a constant which can be positive or negative for Coulomb interaction, but which is invariably positive for gravitational interaction.

The force F can be expressed as a partial derivative of the energy E governing the interaction, with respect to the separation distance x. Thus

$$F = -\partial E/\partial x. \tag{2}$$

This also assures that action balances reaction, there being a mutual force of strength F asserted on A and B.

Introducing r, this becomes

$$F = -\frac{\partial}{\partial x}\int_0^\infty \left(\frac{\partial E}{\partial r}\right) \mathrm{d}r. \tag{3}$$

Our problem then is to determine the general solution for $f(r)$ which brings (3) into accord with (1). Note that $f(r)$ is a function of both r and x, and so can be regarded as a function of r or x taken together with terms in x/r or terms in r/x. The energy E under consideration is finite, for a finite value of x. Hence we can expect $f(r)$ to be represented by a convergent power series in x/r when x is smaller than r or a convergent power series in r/x when r is smaller than x. This follows from Maclaurin's theorem. It may then be shown that a general solution bringing (1) and (3) into conformity is

$$f(r) = \frac{\partial E}{\partial r} = \frac{1}{x^2}\sum \alpha_n (r/x)^n \tag{4}$$

where the summation applies to all integral values of n. The coefficients α_n then need to be determined.

With $n = 0$ or $n = -1$ we must have $\alpha_n = 0$, because the integral in (3) would otherwise be infinite. Similarly, to avoid infinite terms in (4), we must have $\alpha_n = 0$ for $n < -1$ over the range $0 < r < x$ and for $n > 0$ over the range $x < r < \infty$.

3. Physical criteria governing spatial energy distribution

The function $f(r)$ can be restricted by the condition that it is invariable with x beyond a certain distance commensurate with x. This asserts that the force action is local to the energy in the immediate environment of the two interacting bodies. In making this assertion one must depart from the idea embodied in the Mach principle that remote stellar matter determines gravitational interaction locally. Our hypothesis is that force is connected solely with the interaction energy local to the two interacting bodies.

Formulating the above condition:

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial r}\right)=0 \tag{5}$$

for all r, when r is appreciably greater than x.

Restricting ourselves to this latter range, we combine (4) and (5) to obtain

$$\frac{\partial}{\partial x}\left(\frac{1}{x^2}\sum \alpha_n (r/x)^n\right)=-\frac{1}{x^3}\sum \alpha_n (n+2)(r/x)^n=0. \tag{6}$$

Evidently n is -2 and all other α_n values applicable for r greater than x must be zero.

This result is consistent with an argument that the applicable law of force must be dependent upon processes which somehow correlate with the symmetry of space. For example, a symmetrical radial emission from a point source has an intensity which diminishes inversely as the square of distance from the source. Such an action suggests that only one term in n would apply in a general formulation such as (4), as it seems unlikely that a multiplicity of physical actions could be involved in the same region of space, each action having a different symmetry connection and corresponding to a different term in n. Probably, therefore, the expression $f(r)$ will contain but a single $(r/x)^n$ term over a given region of space, as we find for $x<r<\infty$.

Logically, one would not expect a physical action to change abruptly at some arbitrary distance from a reference body. If the form of $f(r)$ changes, it should be at a distance governed by the parameters of the basic two-body system. This implies but one transition set by the separation distance x. There is no other distance parameter in the problem specified. Hence, we next seek to determine n for the unique term $(r/x)^n$ applicable for $0<r<x$.

Since we are considering a spatial energy distribution it is helpful at this stage to examine two possibilities. Firstly, case (a) for which the interaction energy tends to be as close as possible to either body as reference and, secondly, case (b) for which the interaction energy tends to be as remote as possible from either body as reference. This is of interest in as much as there are two basic inverse-square laws, Newton's law of gravitation and Coulomb's law, and there is a physical difference which poses questions. With gravitation like bodies mutually attract and with electric interaction like bodies mutually repel.

3.1. Solution for case (a) interaction

From (4) and the above argument, the energy distribution $f(r)$ is proportional to a single term $(r/x)^n$ over the range $0<r<x$ and to a single term $(r/x)^{-2}$ for $r>x$. Since the energy has to be as close to the reference body as possible, n must be as low as possible. We know that n is a positive integer. Hence $n=1$. The resulting spatial energy distribution is shown in figure 1. The figure is drawn to avoid a discontinuity in the energy spectrum at $r=x$, thereby defining a unique solution.

It will be shown that there are reasons for identifying this particular energy distribution with gravitational interaction.

3.2. Solution for case (b) interaction

This requires the interaction energy to be as remote from the reference body as possible. This is the case for the unique solution that there is no energy at all over the

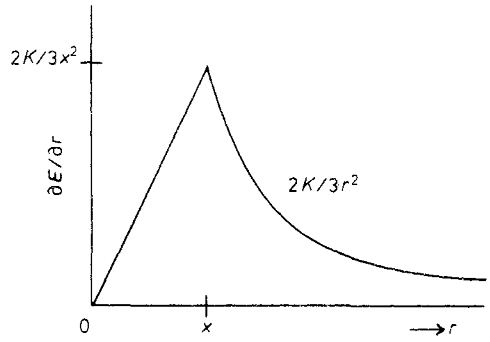

Figure 1. Case (*a*) interaction (gravitational form).

range $0<r<x$, the interaction energy all being confined to the space beyond $r=x$, where the term $(r/x)^{-2}$ applies. The resulting energy distribution is shown in figure 2. Note that we are considering energy averaged over concentric shells of space centred on a reference body, and not specific interaction energy densities which do have finite positive or negative values within the $0<r<x$ range.

This particular energy distribution may be readily identified with the Coulomb interaction by analysis based upon classical field theory.

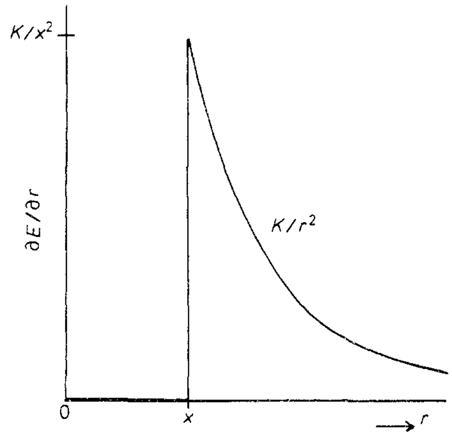

Figure 2. Case (*b*) interaction (Coulomb form).

4. Laws of motion

So long as the two bodies are relatively at rest and their interaction energy is fixed relative to the bodies, it matters not whether the energy distribution has the form set by case (*a*) or that set by case (*b*). Should x change, meaning that the bodies move relative to one another, then there is a deployment of energy between the self-energy of the bodies and the interaction energy. Kinetic energy is exchanged for gravitational energy, for example. The figures indicate that in both cases (*a*) and (*b*) the interaction energy does not change beyond x. Any increase in E is exclusively at x or between 0 and x. Energy shed by the reference body and fed to the interaction has, in case (b), to traverse, on average, the distance x to reach the field. For case (a) the distance is, on average, $\frac{2}{3}x$.

Now, we have argued that the interaction energy E governs the mutual forces between the two bodies. If the bodies are in motion then some energy must be in transit at a finite speed, and is unlikely to be effective in contributing to the force. The result is a retardation of the force action due to this dynamic adjustment. The quantum mechanism of the photon or its gravitational equivalent could well play a part in this process. The point of interest, however, is that it does matter whether we have a case (a) situation or a case (b) situation, because the retardation effects differ for the two cases. The photon energy traverses different distances.

When two interacting bodies move relative to one another they both experience an exchange interaction with the spatially distributed energy E. A photon travels to or from each of them to convey an energy quantum drawn collectively from all parts of nearby space as viewed by that body, a view that is in accord with the energy distribution shown in figures 1 or 2. If each photon has a characteristic propagation speed, c, then the mean distance of energy transfer will be a measure of the retardation of the force action involved. There is one basic question that we need to address. Since the photon is not travelling directly between the bodies, but travels between the body and surrounding space, two photon transfers, one between each body and surrounding space, are needed to assure interaction between the bodies. Are these transfers simultaneous or sequential? If they occur simultaneously, how does body A know that it should move when body B moves? Space is a seat of the energy fluctuations which permit the interaction between the bodies. It is a buffer in the process and, as such, time is needed to assure a sequential reaction. The photon transfer cannot be simultaneous. Thus, in computing the retardation effect upon the law of motion, we will allow for this dual journey.

We proceed to examine how Newton's law of gravitation is modified for a state of motion according to the energy distribution of case (a).

First, it is noted that a perturbation relating energy and retardation involves the square of the retardation time. In case (a) the retardation time varies for different energy elements because they travel different distances. Accordingly, the effective retardation is the root mean square of the time for dual photon exchange. Let T be the total effective retardation. Then $\frac{1}{2}T$ is found by root-mean-square averaging based on figure 1. Thus

$$(\tfrac{1}{2}T)^2 \int_0^x f(r)\,\mathrm{d}r = \int_0^x (r/c)^2 f(r)\,\mathrm{d}r. \tag{7}$$

From figure 1 we know that $f(r)$ is proportional to r over this range of integration. Hence we can solve equation (7) to find

$$T^2 = 2(x/c)^2. \tag{8}$$

The most direct way in which to assess the effects of such a retardation for the sun and planet system is to calculate the associated gravitational energy deficit, that is the amount of energy in transit and so ineffective in asserting force on the planet.

The centrifugal acceleration f of the planet is v^2/x, where v is the orbital velocity in a circular orbit of radius x. This acceleration acting for the retardation period T gives a measure of the displacement under the central gravitational force corresponding to the deficit energy quantity

$$(GMm/x^2)(\tfrac{1}{2}fT^2). \tag{9}$$

Here G is the constant of gravitation, M is the mass of the sun and m is the mass of the planet. Newton's law of gravitation has been used in deriving this result. Now put f as v^2/x and substitute T from (8). The energy deficit given by (9) becomes

$$(GMm/x)(v/c)^2. \tag{10}$$

The quantity $-GMm/x$ is the gravitational potential energy of the system. Therefore, the effect of retardation, if we assume that gravitation involves a law of force conforming with the case (a) situation, is to increase G as it applies in Newton's law of gravitation, effectively by the factor

$$1+(v/c)^2. \tag{11}$$

In terms of force, note that conservation of angular momentum renders v inversely proportional to x, making the v-dependent energy term inversely proportional to x^3. This means that, upon differentiation with respect to x to obtain a force expression using (2), we find the factor in (11) converts to

$$1+3(v/c)^2. \tag{12}$$

Thus in a force equation the value of G needs to be increased by this factor in order to account for retardation effects. This modifies Newton's law of gravitation

$$\mathrm{d}^2u/\mathrm{d}\phi^2+u=GM/h^2 \tag{13}$$

to

$$\mathrm{d}^2u/\mathrm{d}\phi^2+u=GM/h^2+3GM(u^2/c^2). \tag{14}$$

These are laws of motion based on the force relationship and expressed in polar coordinates (u, ϕ). u is $1/x$ and $h=vx$.

Equation (14) is the law of gravitation which emerges from Einstein's general theory of relativity and is, therefore, consistent with the results of that theory based upon this law.

5. Discussion

The idea that Newton's law of gravitation could be modified by retardation effects attributable to the finite speed of propagation of gravitational influence is not new. If energy has to be transferred to convey changes in gravitational action then a modification of the simple law of Newton is to be expected. Gerber (1898), for example, in a paper entitled 'The space and time propagation of gravitation', derived on this basis the same theoretical formula for the anomalous perihelion advance of Mercury as later resulted from Einstein's theory. The correction required gravitation to propagate between sun and planet at the speed of light. However, Gerber's analysis was later challenged and presumably discredited when Einstein's general theory appeared; see the papers by Seelinger (1917a, b) and Oppenheim (1917).

Surdin (1962) has explored the hypothesis that gravitational waves propagated from sun to planet at the speed of light are followed by waves reflected back from planet to sun, thus doubling the retardation action distance and so modifying the law of motion otherwise expected. However, Surdin's method of analysis gave a planetary perihelion advance only two thirds that obtained by Einstein, this being based upon the theory of retarded potential.

In contrast, it has been shown in this paper that the gravitational action cannot be a simple action propagated directly between the interacting bodies. The spatial energy distribution suggests that there is energy transfer to and from the surrounding space or field system. Einstein's formula has emerged without any assumptions of the kind made in conventional field theory, such as that of retarded potentials.

On the other hand, certain assumptions are probably inherent in the method of analysis presented, and these deserve exploration. However, it is felt that the above approach is heuristically reasonable.

It is hoped by further research to examine the electrodynamic implications of the spatial energy distribution for the Coulomb interaction. Preliminary results indicate that the convention (Darwin 1920) of regarding electrodynamic interaction as comprising a first component involving magnetic vector potentials and a second component involving retarded electric potentials could be erroneous. It is difficult to reconcile the latter with the spatial energy distribution presented in figure 2. An analysis of the spatial distribution of magnetic field energy has been made (Eagles and Aspden 1980), but the weakness of this analysis is the use of field theory as the starting point. In contrast, we have, in this paper, deliberately avoided the formulae of field theory and taken the empirical law of force between the interacting bodies as the starting point. Had we used electric field theory, the derivation of the spatial energy distribution shown in figure 2 would be quite straightforward (Aspden 1979). What is now needed is analysis of the electrodynamic interaction, based upon the empirical law of force. The difficulty here is that there is no sure knowledge of this law for interaction between isolated charges in motion. This has been well discussed in the literature, notably by Moon and Spencer (1954), who have contended that relativistic electrodynamics leads us away from accepted magnetic field theory in treating this problem.

It is hoped, nevertheless, that the approach advanced in this paper, of examining the spatial distribution of energy required by our theories, will shed some further light on this important subject and perhaps help to unify the theories of gravitational and electrodynamic interactions. See also the discussion chapter in Aspden (1980).

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